Interaction Terms

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Intro to Interaction Terms

- The use of interaction terms in applied political science research is quite common
- Unfortunately, the misuse of interaction terms is nearly as common
- We need to understand:
    - What is an interaction (or multiplicative term)?
    - What are some of the most common problems with the use of interaction terms?
    - How do we properly model and interpret interaction terms?

Intro to Interaction Terms

- As with most things, theory is a good guide for when to use interaction terms
- If we are going to model an interaction term, some key points to remember:
  1. Include all constitutive terms
  2. Do not incorrectly interpret constitutive terms
  3. Coefficient estimates for interaction terms rarely tell the whole story

Why Use Interaction Terms

- Assume that rather than the direct effect of X_1 on Y, you are interested in some *conditional* relationship
  - Perhaps you hypothesize that X_1 will have an impact on Y when some condition is absent but not if that condition is present
  - Or you hypothesize that the impact of X_1 on Y will be different across a set of conditions

Why Use Interaction Terms

- Generally, anytime we are interested in conditional relationship, then we must model an interaction term to properly test our hypotheses
- Failure to do so would result in an underspecified model unless we account for this in some other way (e.g. subsetting our data).

Why Use Interaction Terms

Always follow theory!

Problem 1

Excluding Constitutive Terms

Assume the following (correctly specified) regression equation:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

- Even if we are only interested in the interaction $X_{1i}X_{2i}$, we still **MUST** include both constitutive terms X_{1i} and X_{2i}
- For example, modeling the above as simply $Y_i = \beta_0 + \beta_3 X_{1i} X_{2i} + u_i$ would be incorrecly specified

Problem 2

Interpreting Constitutive Terms

Given the correct specification::

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

- We must remember that we cannot interpret the coefficient on the constitutive terms as unconditional effect
- In the above example, we cannot interpret β_1 as the effect of X_{1i} on Y_i

Problem 3

Limits of Interpreting Coefficients

- Even when properly specified, interpreting the coefficient on interaction terms can be less than straightforward
- The best way to present meaningful quantities of interest from interactive models is through graphs

Modeling Interaction Effects

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i$$

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} + (\beta_1 + \beta_3 X_{2i}) X_{1i} \\ &= \beta_0 + \beta_2 X_{2i} + \psi_1 X_{1i} \end{aligned}$$

where $\psi_1 = \beta_1 + \beta_3 X_{2i}$. This means:

$$\frac{\partial \mathrm{E}(Y_i)}{\partial X_1} = \beta_1 + \beta_3 X_{2i}.$$

Modeling Interaction Effects

Similarly:

$$\begin{aligned} E(Y_i) &= \beta_0 + \beta_1 X_{1i} + (\beta_2 + \beta_3 X_{1i}) X_{2i} \\ &= \beta_0 + \beta_1 X_{1i} + \psi_2 X_{2i} \end{aligned}$$

which implies:

$$\frac{\partial E(Y_i)}{\partial X_2} = \beta_2 + \beta_3 X_{1i}.$$

"Direct Effects"

If $X_2 = 0$, then:

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1 X_{1i} + \beta_2(0) + \beta_3 X_{1i}(0) \\ &= \beta_0 + \beta_1 X_{1i}. \end{aligned}$$

Similarly, for $X_1 = 0$:

$$\begin{aligned} \mathrm{E}(Y_i) &= \beta_0 + \beta_1(0) + \beta_2 X_{2i} + \beta_3(0) X_{2i} \\ &= \beta_0 + \beta_2 X_{2i} \end{aligned}$$

Types of Interactions: Dichotomous Xs

For

$$Y_i = \beta_0 + \beta_1 D_{1i} + \beta_2 D_{2i} + \beta_3 D_{1i} D_{2i} + u_i$$

we have:

$$\begin{aligned}
\text{E}(Y|D_1 = 0, D_2 = 0) &= \beta_0 \\
\text{E}(Y|D_1 = 1, D_2 = 0) &= \beta_0 + \beta_1 \\
\text{E}(Y|D_1 = 0, D_2 = 1) &= \beta_0 + \beta_2 \\
\text{E}(Y|D_1 = 1, D_2 = 1) &= \beta_0 + \beta_1 + \beta_2 + \beta_3
\end{aligned}$$

Dichotomous and Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 D_i + \beta_3 X_i D_i + u_i$$

gives:

$$\mathrm{E}(Y|X, D = 0) = \beta_0 + \beta_1 X$$
$$\mathrm{E}(Y|X, D = 1) = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X$$

Four possibilities:

- $\beta_2 = \beta_3 = 0$
- $\beta_2 \neq 0$ and $\beta_3 = 0$
- $\beta_2 = 0$ and $\beta_3 \neq 0$
- $\beta_2 \neq 0$ and $\beta_3 \neq 0$

Two Continuous Xs

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + u_i.$$

Implies

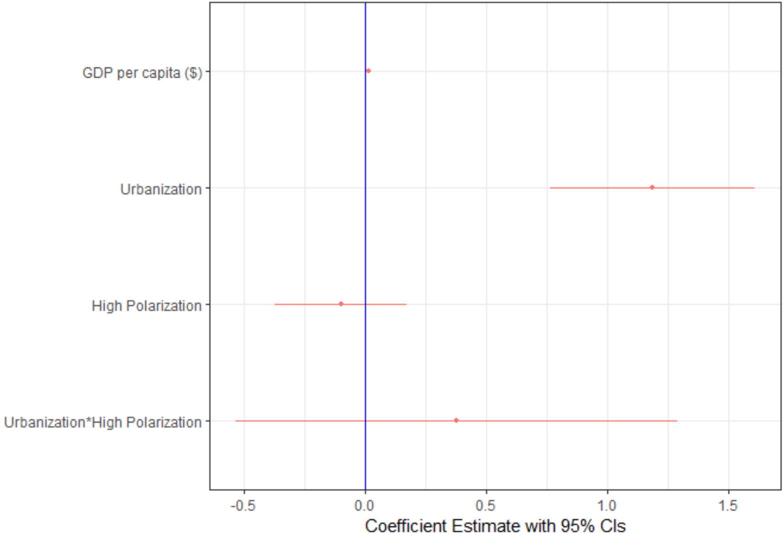
$$\beta_3 = 0 \ \rightarrow \ \frac{\partial E(Y)}{\partial X_1} = \beta_1 \, \forall \, X_2 \text{ and } \frac{\partial E(Y)}{\partial X_2} = \beta_2 \, \forall \, X_1$$

Toy Model

```r
## Load your data ----
# We are using V-Dem version 12
my_data <- readRDS("data/vdem12.rds")

# Let's change names of some of these variables for the sake of simplicity
# I am also subsetting it to only US
us_data <- my_data |>
  filter(country_name == "United States of America") |>
  rename(democracy = v2x_polyarchy,
         gdp_per_capita = e_gdppc,
         urbanization = e_miurbani,
         polarization = v2cacamps,
         polarization_ordinal = v2cacamps_ord) |>
  mutate(high_polarization = ifelse(polarization >= -1, 1, 0))
```

Two-way interaction in R

```
## Model with interaction ----

# We use * (star) between two interaction terms to make sure that each term is
# included individually

# These two are the same
lm(democracy ~ gdp_per_capita + urbanization*as.factor(high_polarization),
   data = us_data)

lm(democracy ~ gdp_per_capita + urbanization + as.factor(high_polarization) +
     urbanization:as.factor(high_polarization),
   data = us_data)
```


GDP per capita ($)
Urbanization
High Polarization
Urbanization*High Polarization
-0.5
0.0
0.5
1.0
1.5
Coefficient Estimate with 95% CIs

Additive vs Interaction Model

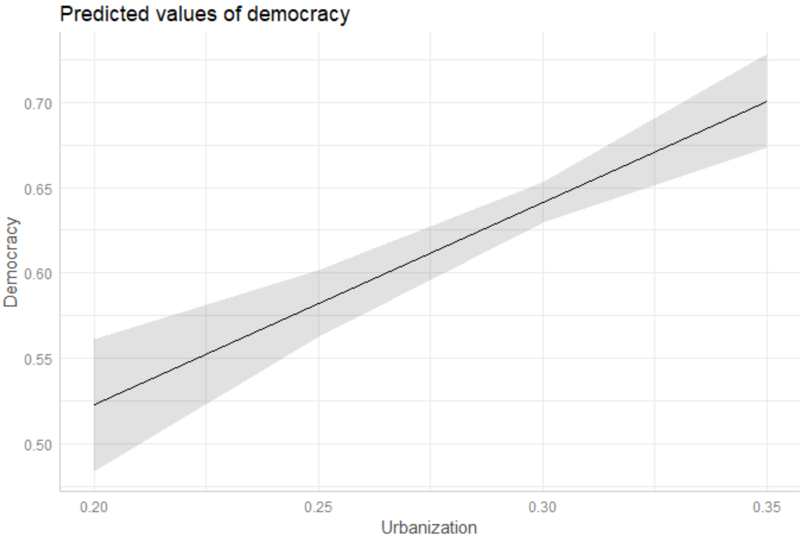
	Dependent variable: democracy	
	Additive model	Interaction model
	(1)	(2)
GDP per capita	0.012***	0.012***
	(0.0004)	(0.0004)
Urbanization	1.284***	1.189***
	(0.178)	(0.213)
High polarization	0.012	-0.100
	(0.009)	(0.137)
Urbanization*High Polarization		0.377
		(0.460)
Constant	0.003	0.028
	(0.051)	(0.059)
Observations	101	101
R2	0.946	0.947
Adjusted R2	0.945	0.944
Residual Std. Error	0.037 (df = 97)	0.038 (df = 96)
F Statistic	569.633*** (df = 3; 97)	425.944*** (df = 4; 96)
Note:		*p<0.1; **p<0.05; ***p<0.01

Predicted values of democracy

```r
## Predicted values ----

# We are going to use `ggeffects` package but there are other different
# packages out there. I prefer this because it works well with ggplot2.
# ggpredict() computes predicted values for all possible levels and values
# from a model's predictors.

# Let's see the predicted values of democracy by different values of urbanization
ggpredict(my_model, terms = c("urbanization"))

# You can just plot these predicted values with plot() function
ggpredict(my_model, terms = c("urbanization")) |> plot() +
  labs(x = "Urbanization", y = "Democracy")
```

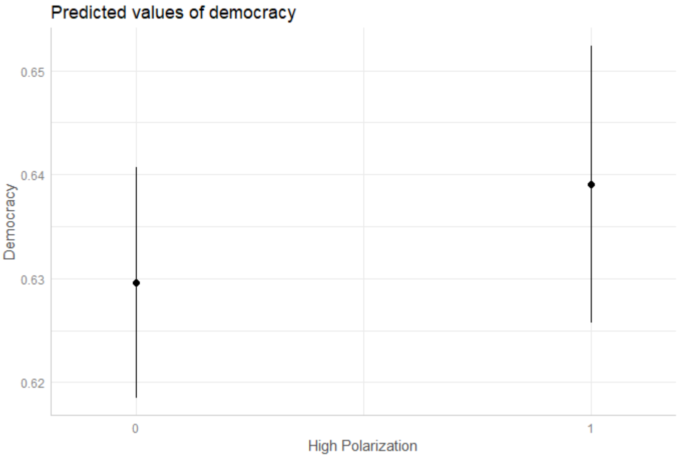
```
> ggpredict(my_model, terms = c("urbanization"))
# Predicted values of democracy

urbanization | Predicted |       95% CI
----------------------------------------
        0.20 |      0.52 | [0.48, 0.56]
        0.25 |      0.58 | [0.56, 0.60]
        0.30 |      0.64 | [0.63, 0.65]
        0.35 |      0.70 | [0.67, 0.73]

Adjusted for:
*    gdp_per_capita = 21.30
* high_polarization =     0
> 
```

Plotting the predicted values



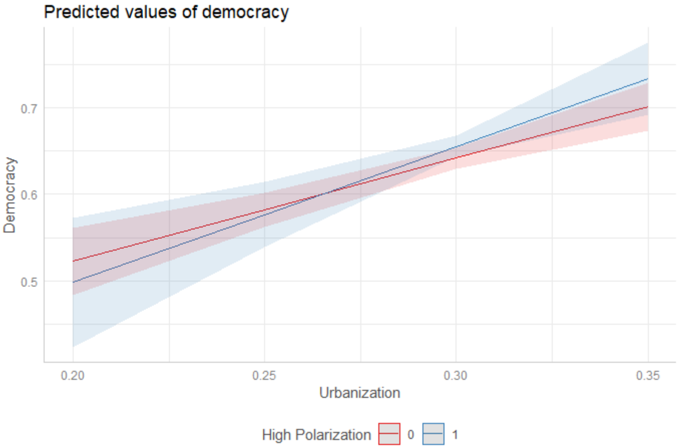
Let's plot for categorical variable

```r
# Let's see the predicted values of democracy by high polarization
ggpredict(my_model, terms = c("high_polarization")) |>
  plot() +
  labs(x = "High Polarization", y = "Democracy")
```



Let's plot the interaction terms

```
# Let's see the predicted values of democracy by urbanization and polarization
ggpredict(my_model, terms = c("urbanization", "high_polarization")) |>
  plot() +
  labs(x = "Urbanization", y = "Democracy", color = "High Polarization") +
  theme(legend.position = "bottom")
```



margins package

```
## Marginal effects ----
# Stata's margins command is very simple and intuitive to use. This package
# helps us port the functionality of Stata's command.

# margins provides “marginal effects” summaries of models and prediction provides
# unit-specific and sample average predictions from models. Marginal effects are
# partial derivatives of the regression equation with respect to each variable in
# the model for each unit in the data; average marginal effects are simply the
# mean of these unit-specific partial derivatives over some sample.

# Let's see margins first
# Warning: margins() command can take a long time depending on the model.
my_margins <- margins(my_model)
summary(my_margins)
```

```
> summary(my_margins)
            factor    AME     SE       z      p   lower  upper
    gdp_per_capita 0.0121 0.0004 29.0552 0.0000  0.0112 0.0129
 high_polarization1 0.0095 0.0097  0.9751 0.3295 -0.0096 0.0285
      urbanization 1.3643 0.2028  6.7278 0.0000  0.9669 1.7618
>
```

Plotting with *margins*